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The Firing Probability of Coupling Spark Gaps of the Multiplying Circuit according to Marx

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1. General

The firing of the coupling spark gaps (CSG) of the multiplying circuit according to Marx (Fig. 1) is effected by the solicitation or the release of only one spark gap. Experience has shown that the other spark gaps fire also successively, spontaneously so to say. These phenomena have been explained in a former paper [1] 1) by means of an equivalent diagram. Nevertheless, only the transient phenomena due to the firing of the first spark gap have been studied at that time, from the theoretical as well as the practical points of view and it has been shown that these phenomena provoked the firing of the next spark gaps. The results of some tests are given hereafter; these tests give an outlook on the firing mechanism in the Marx multiplying circuit from the firing of the first spark gap up to the firing of the last one.

2. Measuring Method

It has been established in earlier papers that the firing of the spark gaps is caused by overvoltages. Nevertheless, the value as well as the duration of these overvoltages are unknown for all the spark gaps with the exception of the first one which fires because its disruptive voltage is exceeded and of the second one, the voltage of which has been evaluated and measured in [1]. The temporal registration of these overvoltages could give a direct outlook on the transient phenomena but it would also require a very bulky equipment. Therefore, an indirect measuring method has been adopted in order to reduce the cost of this equipment. This method, in spite of its simplicity, provides a good overall outlook on the multiplying circuit. The following points must be considered, concerning the measuring method:

Talking about overvoltages in connection with the firing of coupling spark gaps (CSG), voltages exceeding the charging voltage U_0 are meant. This charging voltage corresponds to a gap length s_{stat} which is indicated on the calibration tables concerning sphere gaps. When a given overvoltage is super-imposed to the voltage U_0, the spark gap fires with a known probability, even if the gap length is greater than the value s_{stat}; as a matter of fact, when the gap length of a spark gap regularly irradiated with ultra-violet light and submitted to voltage impulses of constant amplitude and form is varied, the firing probability varies accordingly. The firing probability is zero for great gap lengths, varies between zero and 1 when the gap length is diminished and finally remains practically equal to 1 when the gap length is less than a certain value s_x. This value will be called from now on in this paper "critical gap length". The difference $s_x–s_{stat}$ is a characteristic of the efficient overvoltage on the considered spark gap. Of course, this is only an integral notion, comprising the value as well as the duration of the overvoltage. It is not possible to determine, only with the value $s_x–s_{stat}$, if the spark gap will fire at a high overvoltage with short duration or at a lower overvoltage with longer duration. In spite of this important restriction, it is possible to obtain an outlook of the multiplying circuit mechanism only by studying and analysing the difference $s_x–s_{stat}$ of the various spark gaps.

The critical gap length s_x is determined as follows: The spark gap f_1 in the first stage (Fig. 1) is adjusted to any length s_1. The gap length of the other spark gaps will be much greater than s_1 and thus only f_1 will fire. The firing probability is then determined for the second spark gap f_2, as a function of the gap length. From the graphics, it is possible to determine the gap length at which f_2 fires with a probability of 99.98 %. The second spark gap is then adjusted to a gap length slightly reduced to the one shown in the graphics and corresponding to a 99.98 % firing probability. Thus, after the firing of the first spark gap, the second spark gap will fire with certainty. Afterwards, the firing probability of the third spark gap f_3 is determined when f_1 and f_2 fire with certainty. From the firing probability curve, it is possible to find the value at which f_3 fires surely, and so on. In order to evaluate the firing probability at a

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1) See literature at the end of the paper.

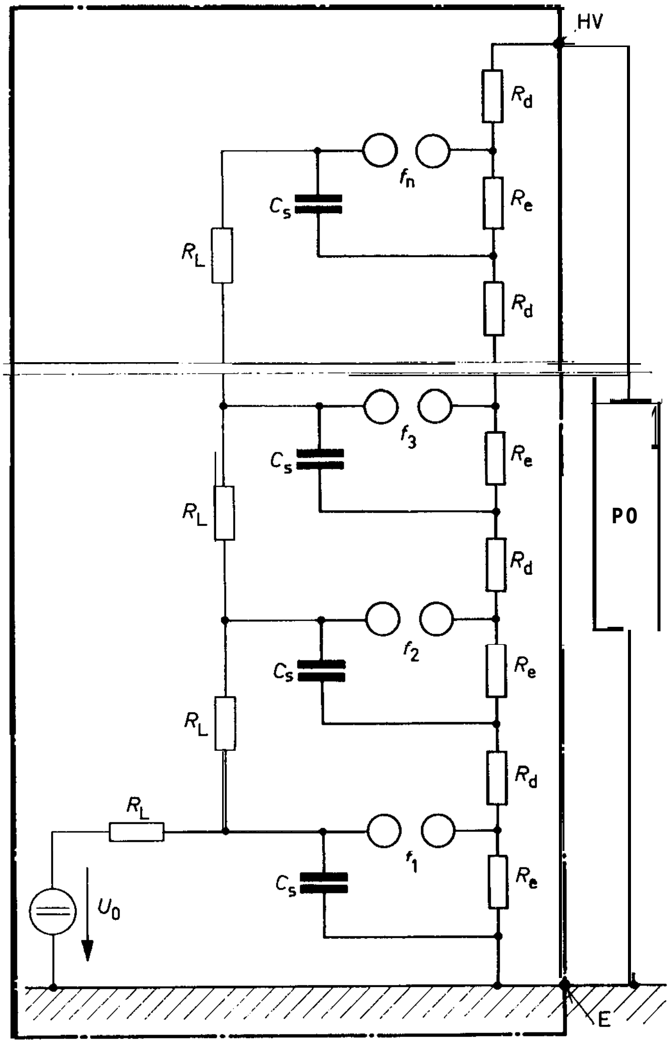

Fig. 1
Schematic Diagram of an n-stage Multiplying Circuit according to Marx
$f_1, f_2 \dots f_n$ coupling spark gaps (CSG) ; C_s impulse capacitor; E earth terminal of generator; **HV** high voltage terminal; **PO** test object; R_d damping resistor; R_e discharging resistor; R_L charging resistor; U_0 charging voltage

given gap length, 50 shots have been effected in each case.

Most of the measurements described hereafter have been effected on a small 6-stage impulse generator, especially designed for this purpose. It has a total charging voltage of 72 kV and an energy of 17 Ws. The tests have been effected as under [1] without damping resistor R_d, in order to reduce the number of the parameters to be considered. In order to complete the results obtained with this relatively small generator, some results issued from measurements on a very large 20-stage generator with a total charging voltage of 4 MV and 300 kWs energy were also examined.

3. Influence of the Load

3.1. Short Circuit and Capacitive Load

The firing probability of the coupling spark gaps in the short-circuited 72-kV generator as a function of the gap length are indicated on Fig. 2: spark gap f_1 is adjusted at 2.0 mm and fires at a voltage of (8.10 ± 0.03) kV. Fig. 2 shows that spark gap f_2 fires with a probability of 1 % at a gap length of $s_2 = 2.85$ mm. If this length is decreased by only 0.1 mm, the firing probability increases to 90 %. Fig. 2 also indicates that spark gap f_2 fires certainly when the gap length is adjusted at 2.65 mm. This value is the critical gap length $s_{x,2}$.

For spark gaps f_3 to f_6 also, gap length variations of approximately 0.2 to 0.3 mm are sufficient to bring the firing probability from 0 % to 100 %. However, the critical gap length $s_{x,k}$ increases with the number of stages k. Fig. 3a shows the critical gap lengths of the short-circuited generator according to the number of stages. Furthermore, the critical gap lengths for various loads are indicated. The points belonging to the same load are joined by a dotted line in order to show the relation of the measured points.

The fact that the critical gap lengths of the spark gaps depend on the number of stages is indicated in Fig. 3a and can be explained for the short-circuited generator by means of the “final value” of overvoltages determined in [1], as follows:

In paper [1], an equivalent diagram was derived for the transient phenomena resulting from the firing of the first spark gap. This diagram mainly consisted of a resistor chain loaded with longitudinal and transverse resistors. The chain circuit of Fig. 4 is fed. after the firing of the first spark gap,

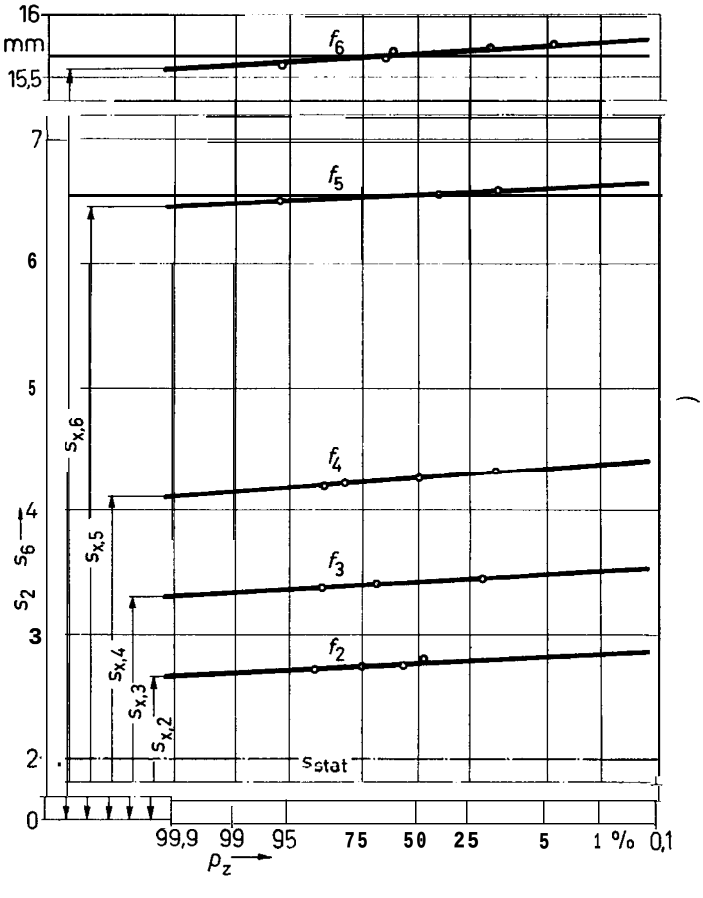

Fig. 2
The Firing Probability of Spark Gaps $f_2 \dots f_6$ in a Short-circuited L-stage Impulse Generator as a Function of the Gap Length
s_k (k = 1, 2, 3...6) gap length of the k-th spark gap; $s_{x,k}$ “critical gap length” of the k-th spark gap; s_{stat} gap length for statical firing voltage of first spark gap; p_z firing probability
other data. see Fig. 1

only by the charged impulse capacitor of the first stage. After the firing of other spark gaps, a greater number of impulse capacitors will operate as supply source for the rest of the chain circuit representing the generator part which has not yet fired. It must be noted that the capacitors C_s discharge on the resistors R_e, in general with time constants of a few 10^{-5} s. The voltages on capacitors C_s can thus be considered as constant, compared with the firing phenomena which take place within less than 10^{-6} s. Therefore, in the earlier paper [1], two components of the overvoltages have been distinguished:

1. One fast part vanishing within approx. 10^{-7} to 10^{-8} s, due to the transient phenomena of the stray capacitances C and *K* (Fig. 4), and
2. One slower part vanishing within a few 10^{-5} s, due to the voltage drop along the discharge resistors R_e at the time of the impulse capacitors discharging.

Earlier, the first 10^{-6} s of the second part have been considered as "final values" because, compared with the extremely short times of the first part, they appear constant during a few 10^{-7} s. Therefore, during approx. 10^{-6} s for instance, the capacitors C_s can be considered as replaced by batteries, without internal resistance, with the corresponding voltage (Fig. 5); as a matter of fact, the large capacitances C_s do not represent a noticeable impedance in the transient phenomena, compared with the stray capacitances. Only their load and inductance are important. This inductive internal resistance will be neglected, nevertheless, for the following reasons: In the example of Fig. 5, after the firing of f_1, the overvoltages on the other spark gaps tend towards a standard value of $^1/_5$, the charging voltage being Uo. When f_2 has fired also, the final value on the not fired spark gaps would be $^2/_4$ and so on. Generally, the final value $\ddot{U}_{end}$ of the overvoltages on the not fired spark gaps in a short-circuited generator of n stages, of which m stages have fired already, is given by the formula

$$\ddot{U}_{end} = \frac{m}{n - m} \qquad (1)$$

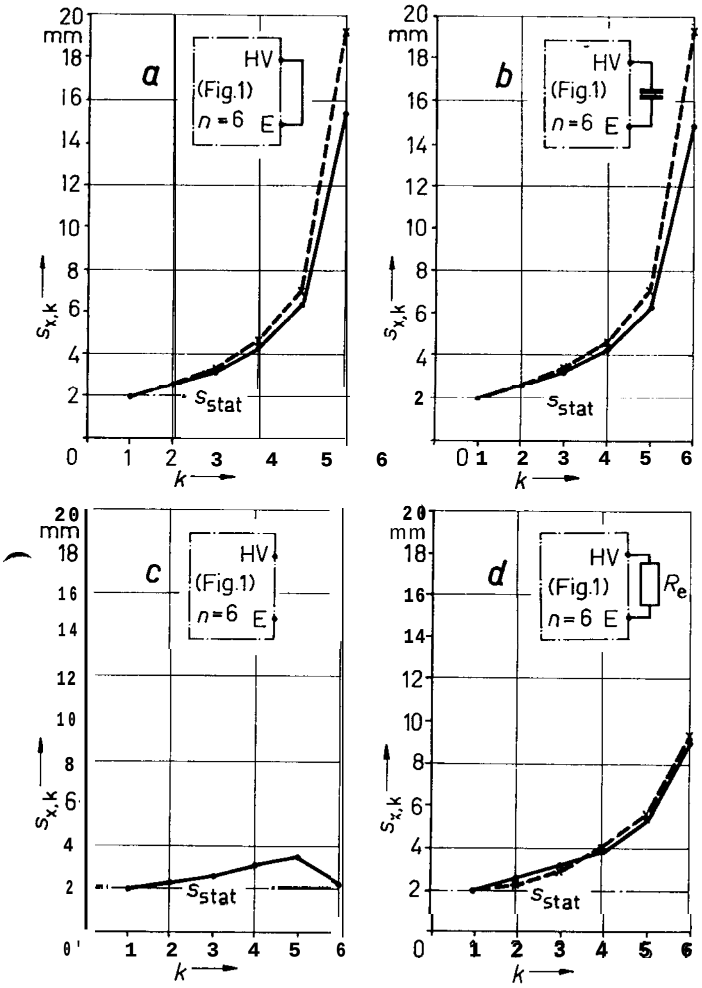

Fig. 3
Critical Gap Length $s_{x,k}$ of a **6-stage** Impulse Generator with Various Load Conditions

a short circuit
b capacitive load
c no-load
d resistive load
k stage number
n number of stages

Points ***HV*** and *E* correspond to the terminals in the circuit of Fig. 1.

———— measured $s_{x,k}$
- - - - - - calculated $s_{x,k}$ according form. (1)

The final values of the overvoltages are thus accessible through calculation. The traced line of the gap length of spark gaps, depending on the number of stages, of the short-circuited generator (Fig. 3), can thus be explained by means only of the simplified formula (1).

Once we admit that the final values $\ddot{U}_{end}$ only cause the firing of the spark gaps, the critical gap length $s_{x,k}$, at which the spark gaps still fire, can be evaluated. The result of this calculation is indicated on Table I for the 6-stage, 72-kV generator. The relation between the firing voltage (column 4) and the gap length (column 5) has been derived from the known calibration curve of a 2 cm 0 sphere gap, for a symmetrical voltage stress. The degree of correspondence between the calculated values (column 5) and the measured ones (column 6) can be considered as satisfactory, taking into consideration the rough approximation in calculation evaluation and the simple use of the spark gap calibration tables, not considering any impulse factors.

Calculation ***Example for the Critical Gap Lengths***

Table I

1	2	3	4	5	6
Fired Spark Gaps	U_{end} of still not fired Spark Gaps	$1 + U_{end}$	$(1 + U_{end})\ U_L$ $(U_L = 8{,}1$ kV$)$ kV	$s_{x,k}$ calculated from column 4 mm	$s_{x,k}$ measured mm
f_1	$^1/_5$	1,2	9,72	$s_{x,2}$ = 2,5	2,65
$f_1 + f_2$	$^2/_4$	1,5	12,16	sx.3 = 3,3	3,20
$f_1 + f_2 + f_3$	$^3/_3$	2,0	16,20	$s_{x,4}$ = 4,6	4,00
$f_1 + f_2 + f_3 + f_4$	$^4/_2$	3,0	24,30	$s_{x,5}$ = 7,1	6,40
$f_1 + f_2 + f_3 + f_4 + f_5$	$^5/_1$	6,0	48,60	$s_{x,6}$ = 19,4	15,50

Nevertheless, we must expect that the approx. calculation from formula (1), for low values of m, gives too low values for the gap lengths $s_{x,k}$, that is, when only a few spark gaps have fired. As a matter of fact, the rapidly changing part of the overvoltages is predominant in the lower stages of the generator, compared with the final values $\ddot{U}_{end}$, and the measured values $s_{x,k}$ are higher than the calculated ones. On the other hand, for high values of *m*, that is when almost all

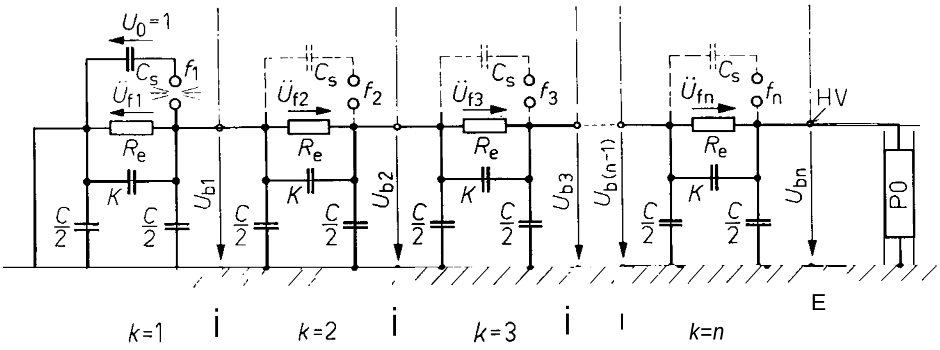

Fig. 4

Equivalent Diagram of Marx Multiplying Circuit for describing the Transient Phenomena after the Firing of the First Spark Gap

$f_1, f_2 \ldots f_n$ coupling spark gaps (CSG); U_b voltage of the discharging resistor chain against earth; $Ü_f$ overvoltage on the spark gap; C stray capacitance against earth; C_s impulse capacitor; E earth terminal of generator; HV high voltage terminal; K stray capacitance between generator stages; PO test object; R_e discharging resistor; U_0 charging voltage; k stage number; n number of stages

spark gaps of the generator have fired, the theoretical values $s_{x,k}$ are rather higher than the measured values because, now, the numerous sources of voltages constituted by the charged capacitors cannot, as it was the case in the theoretical calculation, deliver their energy in the chain circuit of the discharge resistors without taking into consideration the resistive element. The differences between measured and calculated values are very pronounced in the case of the 20-stage generator of 4 MV discussed at the beginning of this paper (Fig. 6).

The results of the measurements on the capacitively loaded generator — which is the most frequent case encountered on the test platform — are little different from those of the short-circuited generator because already a capacitance of approx. 100 pF at the output of the generator offers a negligible resistance to the transient phenomena which take place within a few 10^{-7} s (Fig. 3b). The careful study of the firing probability in case of short circuit, even of no practical significance in the case of impulse tests, has an evident meaning due to the narrow relationship between the firing phenomena in the cases of short circuit and capacitive load.

3.2. No-load Operation

As shown in the earlier paper [1], the overvoltages of a generator in no-load operation are tending to a zero end value. Therefore, the conditions favourable for a firing are present for a short time only. Theoretically, it is expected that the value $s_{x,k}$ is smaller in the case of no-load operation than in the case of short circuit.

As indicated on Fig. 3c, the critical gap lengths $s_{x,k}$, at which the generator still fires, are, as a matter of fact, smaller than in the case of the short-circuited generator. For f_2, the difference between no-load and short circuit operations is still small but it increases considerably on the upper stages. In particular, it must be remarked that the length $s_{x,k}$ is much smaller for the last spark gap than for the preceding one. This phenomenon is probably due to the following causes: An overvoltage on the last spark gap of the generator in no-load operation only exists until the earth capacitance is charged through the discharging resistor connected to the sphere. The earth capacitance of this sphere is very small and therefore the overvoltage is of short duration. When the earth capacitance is raised by only a few pF, the max. gap length of the last spark gap increases also.

3.3. Resistive Load

For this test, the output of the 72-kV generator is connected to the earth electrode through an ohmic resistor of same value as a discharge resistor. After the firing of f_1, the overvoltages on the other spark gaps tend to the value $^1/_6$, the charging voltage being U_0. When f_2 also fires, the end value of overvoltages on the not-fired spark gaps will be $^2/_4$, and so on. The critical values $s_{x,k}$ can be calculated from the end values same as in the case of the short-circuited generator. The degree of correspondence between the calculated values of gap lengths and the measured values is very satisfactory, as shown on Fig. 3d.

4. Influence of some Coupling Elements

With the same method of measurement, it is possible to investigate the influence of all possible parameters on the

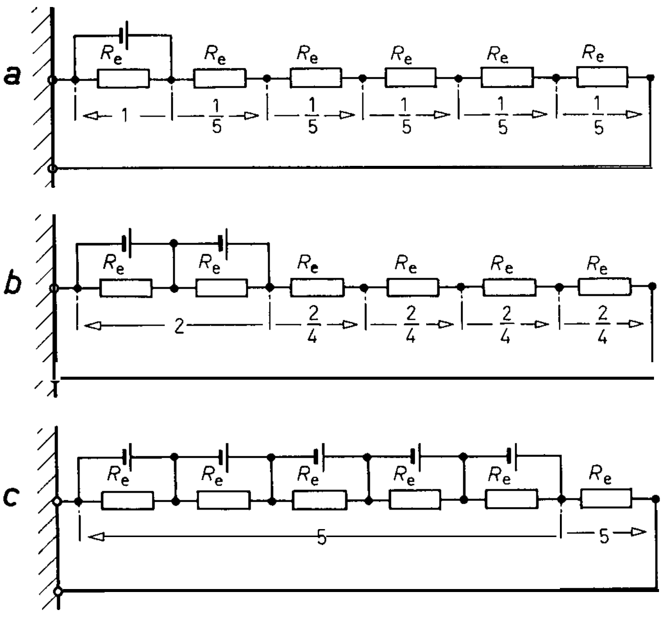

Fig. 5

Calculation of the “Final Value” of Overvoltages in a Short-circuited 6-stage Impulse Generator

a situation after firing of the first spark gap;
b situation after firing of the first and second spark gap;
c situation after firing of the first up to the fifth spark gap
R_e discharging resistor

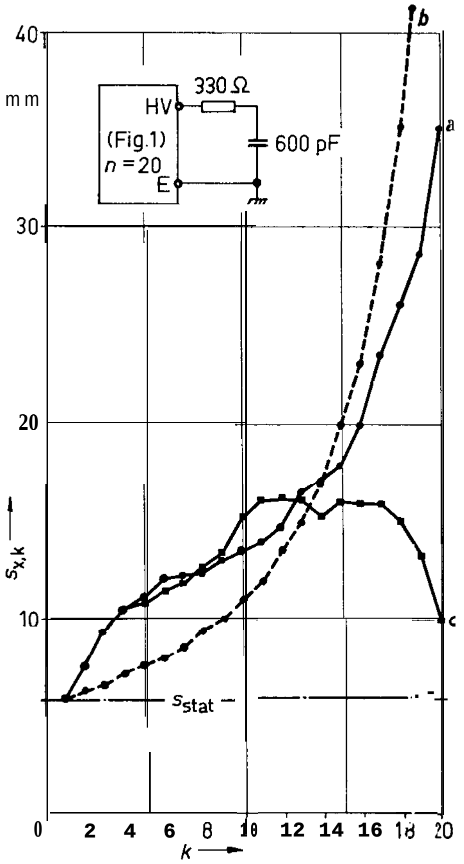

Fig. 6

Critical Gap Lengths $s_{x,k}$ of a 20-stage, 4-MV 300-kWs Generator with Various Load Conditions

a measured critical gap length with a resistive load through a series coupling of R_{dc} = 330 Ω and C_b = 600 pF

b calculated critical gap lengths for the resistive load of 330 Ω

c measured critical gap length of the generator at no-load

k stage number; s_{stat} gap length for statical firing voltage of the first spark gap

firing probability of coupling spark gaps and to draw the conclusions for the design and the operation of impulse generators. Two particularly important influences are mentioned here: Fig. 7 shows the relationship between the overvoltages and the ohmic value of the discharging resistors R_e; this relationship has been measured and calculated in paper [1]. Fig. 8 shows the influence of the stray capacitances C and K upon the voltage grading in the generator and therefore upon the over-voltages.

5. The firing probability of the second spark gap for different irradiation conditions

With the circuit arrangement shown on Fig. 9, the influence of different irradiation conditions on the firing probability of the 72-kV generator coupling spark gaps has been investigated. Direct irradiation of the spark gaps has been prevented by means of the black mat cardboards 1. The spark light from a spark gap which had fired could reach the other spark gaps only through diffuse reflection on the white cardboard 2. The irradiation intensity could be easily modified by changing the position of the reflecting screen. The decrease in intensity of the spark light in the stretch spark gap to screen and screen to not-fired spark gap is due to the two following reasons:

1. The light intensity decreases with the square of the distance from the light source.

2. Part of the light is absorbed in the an. This concerns in particular the ultra violet rays.

In order to evaluate the amount by which the spark light can be decreased without tampering with the perfect firing of the spark gaps, it is sufficient to examine the influence of the spark light of the first spark gap f_1 on the firing probability of f_2. After the static firing of f_1, f_2 has the most unfavourable firing conditions with regards to the other spark gaps, especially in the frequent case of a capacitively loaded or short-circuited generator, because s_{x2} is smaller than the critical gap lengths of all the other spark gaps (Fig. 3a). Therefore, if f_2 does not fire with certainty, the other spark gaps will also definitely not fire.

The results of these measurements are given in Fig. 10. The gap length of f_1 is 2 mm. Curve a gives the firing probability line in case of direct irradiation, curve b shows the case where a black screen f_1 and f_2 prevents direct irradiation and the reflective screen is placed at a distance of 20 cm from the spark gap. For a distance of 70 cm between reflective screen and spark gap, we obtain curve c.

The curves of firing probability of Fig. 10 can be explained as follows: The spark length s_2 is varied while spark gap f_2 is subjected to a voltage of determined shape and amplitude. If, on the other hand, the distance s_2 remains unchanged and the voltage amplitude is varied (its shape remaining constant), the characteristic impulse lines can be determined, i.e. the relationship between firing voltage and

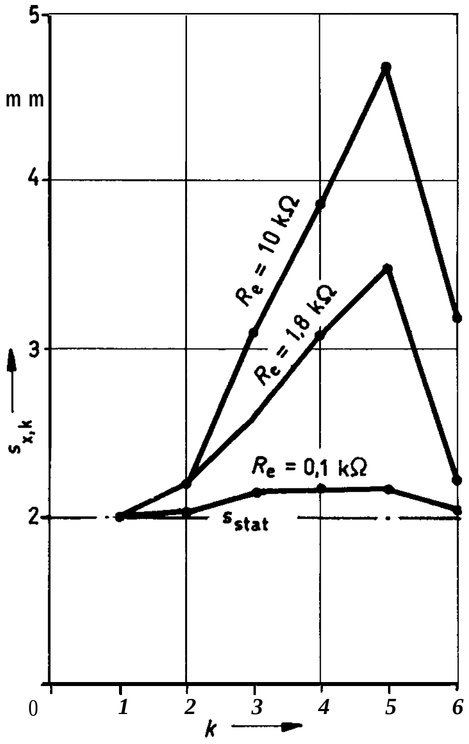

Fig. 7

Critical Gap Length of a 6-stage Generator with Various Discharging Resistors R_e

k stage number; s_{stat} gap length for statical firing voltage of the first spark gap

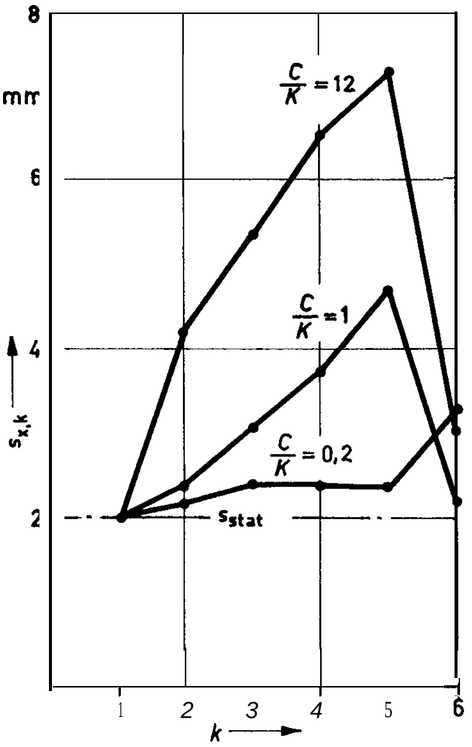

Fig. 8
Critical Gap Lengths of a 6-stage Generator with Various Stray Capacitance Conditions
k stage number; s_{stat} gap length for statical firing voltage of the first spark gap; C stray capacitance against earth; *K* stray capacitance between generator stages

firing time. We obtain as lower limit of a spread band the characteristic line of the firing time, and as upper limit the characteristic line of the dispersion time [2]. The characteristic line of the dispersion time shows the probability of the spark formation. It is therefore particularly influenced by the spark gap irradiation. On the other hand, the characteristic line of the firing time is fixed as long as the voltage shape and gap length remain unchanged. The characteristic line of the firing time is characterized by a zero firing probability while the characteristic line of the dispersion time is characterized by a 100 % firing probability. In a similar way to the usual characteristic lines, the firing probability lines in function of the gap length are indicated on Fig. 9. It can be remarked that the 3 curves *a*, *b* and c converge in the case of decreasing firing probability and for very small probabilities ($<$ 0.1 %) they tend asymptotically towards a given gap length (approx. 2.8 mm). If the tension line of the fired spark gap for this distance is oscillographed and the voltage amplitude as well as the firing time are reported upon the diagram of the characteristic impulse lines, we obtain one point of the firing time characteristic line.

Contrary to what happens in the case of small firing probabilities, the gap lengths at which a high probability is attained ($>$ 99.9 %) are strongly influenced by the irradiation intensity. This is in accordance with the relationship between the dispersion time characteristic line and the irradiation. An important feature must be remarked for the present consideration: curve c (very small irradiation) only reaches a firing probability of approx. 40 % at a gap length of s_2 = 2 mm. As the gap length of the spark gap in the first stage is also 2 mm, it is not advisable to choose for the spark gap of the second stage a length less than 2 mm. The firing probability of 40 % means that the impulse capacitors C_s cannot be connected in series with certainty, due to the low irradiation of the second spark gap.

6. Tests on the 4 MV Generator

The results of the measurements given on Fig. 6, concerning a 20-stage generator with 300-kWs energy, generally confirm the information provided by the small 6-stage generator. For the evaluation of the gap lengths of the spark gaps in the generator loaded by an external damping resistor R_{de} of 330 Ohms and a capacitance of 600 pF, C_b has been considered as a short circuit, that is, only resistor R_{de} has been taken into consideration. The final values of the overvoltages are obtained by analogy to those described under parag. 3.3, by spreading the voltage over the 20 discharging resistors R_e (250 Ohms each) and the R_{de} resistor. The difference between the measured and the calculated values of critical gap lengths $s_{x,k}$ have already been discussed under 3.1.

It is worth mentioning that the critical values $s_{x,k}$ for the generator loaded or short-circuited differ very little from one another up to the tenth stage. It is only in the upper half of the generator that the final values of overvoltages play an essential part; in other words, it is only after the firing of f_{10} that the generator "takes notice" on how it is loaded on the last stage. Before the tenth stage, neither the

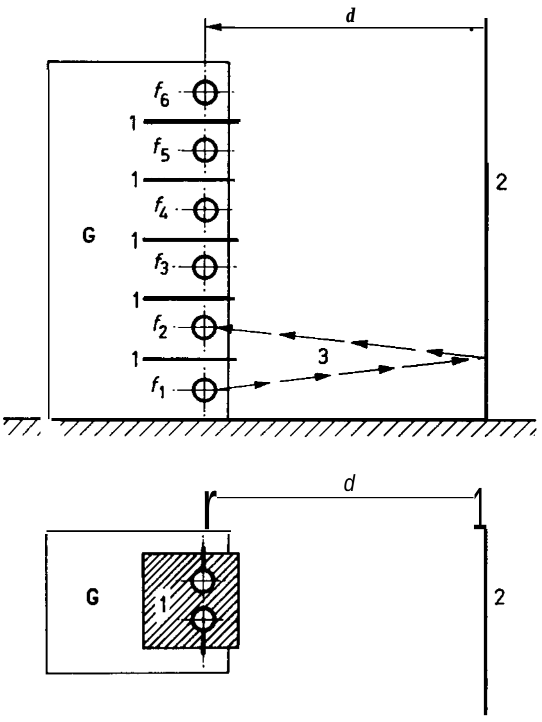

Fig. 9
Test Arrangement for Measuring the Irradiation Influence of the First Spark Gap on the Firing Probability of the Second Spark Gap
d varied distance between spark gap and reflection screen; $f_1, f_2 ... f_6$ coupling spark gaps (CSG); G impulse generator; I black screen preventing direct mutual irradiation of the spark gaps; 2 white reflection screen; 3 indirect light itinerary from first to second spark gap through reflection on screen

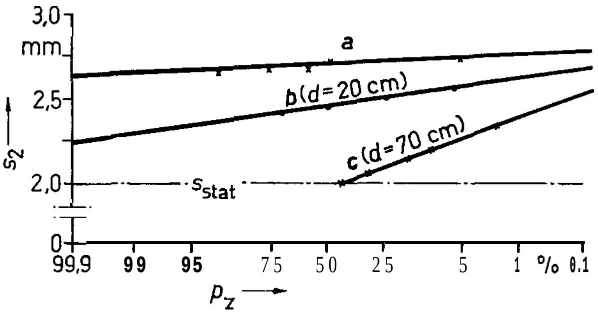

Fig. 10

Firing Probability p_z of the Second Spark Gap in Relation of the Gap Length by Varying Irradiation Conditions

a direct irradiation from first to second spark gap; *b* indirect irradiation through the reflection screen (see Fig. 9); distance of the reflection screen d = 20 cm; *c* indirect irradiation; distance of the reflection screen d = 70 cm; s_{stat} gap length for statical firing voltage of first spark gap

capacitive couplings along the 20-stage generator nor the reduced amplitude of the final values $\ddot{U}_{end}$ can furnish to the first stage some clear information upon the conditions prevailing on the generator top. Otherwise, the critical gap lengths should more or less differ with each charging voltage value, even in the lower half of the generator.

7. Summary

For evaluating the firing probability in some multiplying circuits according to Marx, in function of the gap length, it is sufficient to investigate the mechanism of self-firing of the coupling spark gaps. The influence of important parameters, such as the load, the stray capacitances, the discharging resistors and irradiation, upon the firing of spark gaps can be investigated using a convenient and simple measuring equipment enabling the measuring of the gap lengths and the counting of the firings or non-firings in a series of tests.

Literature

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